Fundamentals of Analyzing and Mining Data Streams

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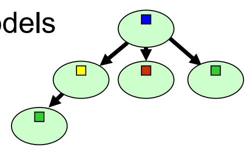
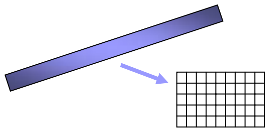
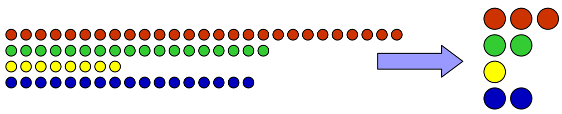
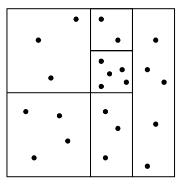
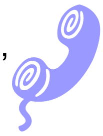
Outline

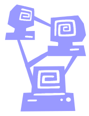
1. Streaming summaries, sketches and samples
   – Motivating examples, applications and models
   – Random sampling: reservoir and minwise
     ▪ Application: Estimating entropy
   – Sketches: Count-Min, AMS, FM
2. Stream Data Mining Algorithms
   – Association Rule Mining
   – Change Detection
   – Clustering

Data is Massive

- Data is growing faster than our ability to store or index it
- There are 3 Billion Telephone Calls in US each day, 30 Billion emails daily, 1 Billion SMS, IMs.
- Scientific data: NASA's observation satellites generate billions of readings each per day.
- IP Network Traffic: up to 1 Billion packets per hour per router. Each ISP has many (hundreds) routers!
- Whole genome sequences for many species now available: each megabytes to gigabytes in size



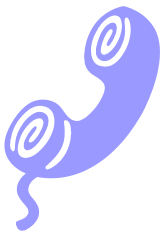


Massive Data Analysis

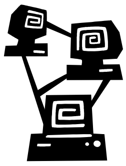
Must analyze this massive data:

- Scientific research (monitor environment, species)
- System management (spot faults, drops, failures)
- Customer research (association rules, new offers)
- For revenue protection (phone fraud, service abuse)

Else, why even measure this data?



Example: Network Data

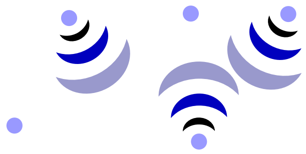


- Networks are sources of massive data: the metadata per hour per router is gigabytes
- Fundamental problem of data stream analysis:
  Too much information to store or transmit
- So process data as it arrives: one pass, small space: the *data stream* approach.
- Approximate answers to many questions are OK, if there are guarantees of result quality

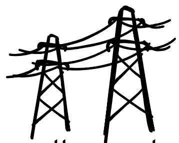
Streaming Data Questions

- Network managers ask questions requiring us to analyze and mine the data:
  - Find hosts with similar usage patterns (clusters)?
  - Which destinations or groups use most bandwidth?
  - Was there a change in traffic distribution overnight?
- Extra complexity comes from limited space and time
- Will introduce solutions for these and other problems

Other Streaming Applications



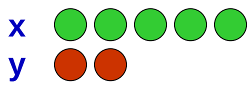
- Sensor networks
  - Monitor habitat and environmental parameters
  - Track many objects, intrusions, trend analysis…



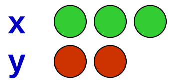
- Utility Companies
  - Monitor power grid, customer usage patterns etc.
  - Alerts and rapid response in case of problems

Data Stream Models

- We model data streams as sequences of simple tuples
- Complexity arises from massive length of streams
- Arrivals only streams:
  - Example: (x, 3), (y, 2), (x, 2) encodes the arrival of 3 copies of item x, 2 copies of y, then 2 copies of x.



  - Could represent eg. packets on a network; power usage
- Arrivals and departures:
  - Example: (x, 3), (y,2), (x, -2) encodes final state of (x, 1), (y, 2).



  - Can represent fluctuating quantities, or measure differences between two distributions

Approximation and Randomization

- Many things are hard to compute exactly over a stream
  - Is the count of all items the same in two different streams?
  - Requires linear space to compute exactly
- Approximation: find an answer correct within some factor
  - Find an answer that is within 10% of correct result
  - More generally, a $(1\pm \varepsilon)$ factor approximation
- Randomization: allow a small probability of failure
  - Answer is correct, except with probability 1 in 10,000
  - More generally, success probability $(1-\delta)$
- Approximation **and** Randomization: (ε, δ)-approximations

Structure

1. Stream summaries, sketches and samples
   - Answer simple distribution agnostic questions about stream
   - Describe properties of the distribution
   - E.g. general shape, item frequencies, frequency moments
2. Data Mining Algorithms
   - Extend existing mining problems to the stream domain
   - Go beyond simple properties to deeper structure
   - Build on sketch, sampling ideas

- Only a representative sample of each topic, many other problems, algorithms and techniques not covered

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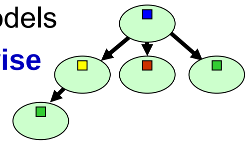
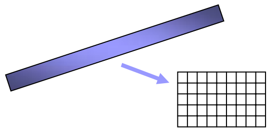
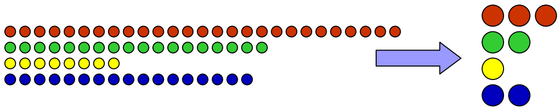
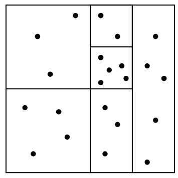
   - Association Rule Mining
   - Change Detection
   - Clustering

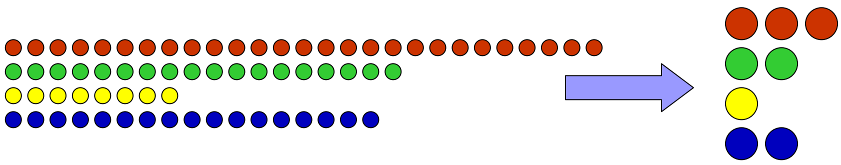






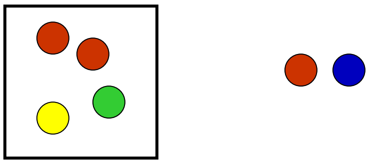


Sampling From a Data Stream



- Fundamental prob: sample m items uniformly from stream
  - Useful: approximate costly computation on small sample
- Challenge: don't know how long stream is
  - So when/how often to sample?
- Two solutions, apply to different situations:
  - Reservoir sampling (dates from 1980s?)
  - Min-wise sampling (dates from 1990s?)

Reservoir Sampling



- Sample first m items
- Choose to sample the i'th item (i>m) with probability m/i
- If sampled, randomly replace a previously sampled item

- Optimization: when i gets large, compute which item will be sampled next, skip over intervening items. [Vitter 85]

Reservoir Sampling - Analysis

- Analyze simple case: sample size m = 1
- Probability i'th item is the sample from stream length n:
  - Prob. i is sampled on arrival × prob. i survives to end

$$\frac{1}{\cancel{i}} \times \frac{\cancel{i}}{\cancel{i+1}} \times \frac{\cancel{i+1}}{\cancel{i+2}} \ldots \frac{\cancel{n-2}}{\cancel{n-1}} \times \frac{\cancel{n-1}}{n}$$

$$= 1/n$$

- Case for m > 1 is similar, easy to show uniform probability
- Drawbacks of reservoir sampling: hard to parallelize

Min-wise Sampling

- For each item, pick a random fraction between 0 and 1
- Store item(s) with the smallest random tag [Nath et al.'04]

- Each item has same chance of least tag, so uniform
- Can run on multiple streams separately, then merge

Application of Sampling: Entropy

- Given a long sequence of characters

  $S = \langle a_1, a_2, a_3 \ldots a_m \rangle$ each $a_j \in \{1 \ldots n\}$
- Let f_i = frequency of i in the sequence
- Compute the empirical entropy:

  $$H(S) = - \sum_i f_i/m \log f_i/m = - \sum_i p_i \log p_i$$

- Example: $S = \langle$ **a, b, a, b, c, a, d, a** $\rangle$
  - $p_a = 1/2$, $p_b = 1/4$, $p_c = 1/8$, $p_d = 1/8$
  - $H(S) = ½ + ¼ \times 2 + 1/8 \times 3 + 1/8 \times 3 = 7/4$
- Entropy promoted for anomaly detection in networks

Sampling Based Algorithm

- Simple estimator:
  - Randomly sample a position j in the stream
  - Count how many times a_j appears subsequently = r
  - Output $X = -(r \log r/m – (r-1) \log(r-1)/m)$
- Claim: Estimator is unbiased – $E[X] = H(S)$
  - Proof: prob of picking $j = 1/m$, sum telescopes correctly
- Variance is not too large – $Var[X] = O(\log^2 m)$
  - Can be proven by bounding $|X| \leq \log m$

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Analysis of Basic Estimator

- A general technique in data streams:
  - Repeat in parallel an unbiased estimator with bounded variance, take average of estimates to improve result
  - Apply Chebyshev bounds to guarantee accuracy
  - Number of repetitions depends on ratio $Var[X]/E^2[X]$
  - For entropy, this means space $O(\log^2 m/H^2(S))$
- Problem for entropy: when $H(S)$ is very small?
  - Space needed for an accurate approx goes as $1/H^2$!

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Outline of Improved Algorithm

- Observation: only way to get $H(S) = o(1)$ is to have only one character with p_i close to 1
  - **aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaabaaaaa**
- If we can identify this character, and make an estimator on stream without this token, can estimate H(S)
- How to identify and remove all in one pass?
- Can do some clever tricks with ‘backup samples’ by adapting the min-wise sampling technique
- Full details and analysis in [Chakrabarti, C, McGregor 07]
  - Total space is $O(\varepsilon^{-2} \log m \log 1/\delta)$ for (ε,δ) approx

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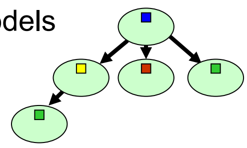
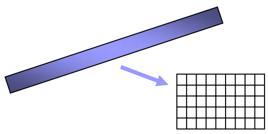
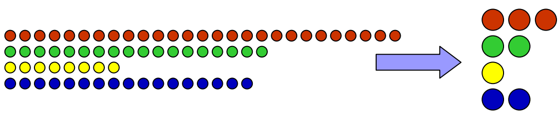
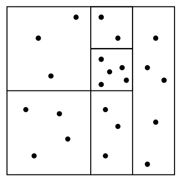


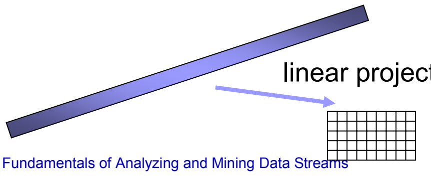




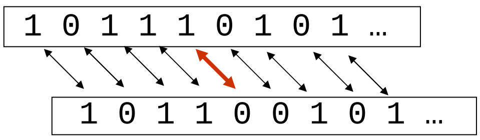


Sketches

- Not every problem can be solved with sampling
  - Example: counting how many distinct items in the stream
  - If a large fraction of items aren't sampled, don't know if they are all same or all different
- Other techniques take advantage that the algorithm can "see" all the data even if it can't "remember" it all
- (To me) a sketch is a linear transform of the input
  - Model stream as defining a vector, sketch is result of multiplying stream vector by an (implicit) matrix



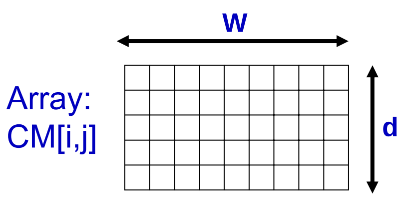
Trivial Example of a Sketch



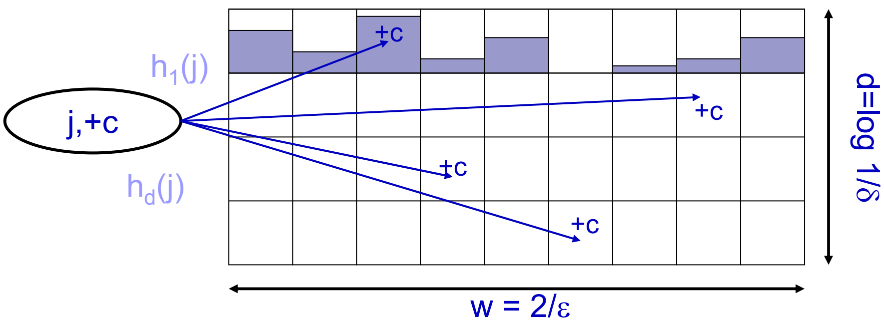
- Test if two (asynchronous) binary streams are equal
  $d_=(x,y) = 0$ iff $x=y$, 1 otherwise
- To test in small space: pick a random hash function h
- Test $h(x)=h(y)$: small chance of false positive, no chance of false negative.
- Compute $h(x)$, $h(y)$ incrementally as new bits arrive (Karp-Rabin: $h(x) = x_i 2^i \bmod p$)
  - Exercise: extend to real valued vectors in update model

Count-Min Sketch

- Simple sketch idea, can be used for as the basis of many different stream mining tasks.
- Model input stream as a vector x of dimension U
- Creates a small summary as an array of $w \times d$ in size
- Use d hash function to map vector entries to [1..w]
- Works on arrivals only and arrivals & departures streams



CM Sketch Structure



- Each entry in vector x is mapped to one bucket per row.
- Merge two sketches by entry-wise summation
- Estimate $x[j]$ by taking $\min_k CM[k,h_k(j)]$
  - Guarantees error less than $\varepsilon||x||_1$ in size $O(1/\varepsilon \log 1/\delta)$
  - Probability of more error is less than $1-\delta$

[C, Muthukrishnan '04]

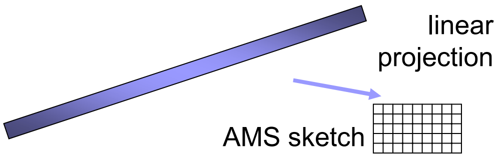
Approximation

Approximate $x'[j] = \min_k CM[k,h_k(j)]$

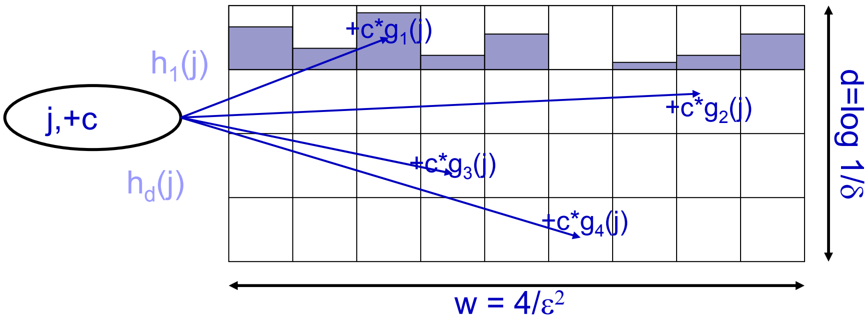
- Analysis: In k'th row, $CM[k,h_k(j)] = x[j] + X_{k,j}$
  - $X_{k,j} = \Sigma\ x[i] \mid h_k(i) = h_k(j)$
  - $E(X_{k,j}) = \Sigma\ x[k]^*Pr[h_k(i)=h_k(j)]$
    $\leq Pr[h_k(i)=h_k(k)]\ ^*\ \Sigma\ a[i]$
    $= \varepsilon\ ||x||_1/2$ by pairwise independence of h
  - $Pr[X_{k,j} \geq \varepsilon||x||_1] = Pr[X_{k,j} \geq 2E(X_{k,j})] \leq 1/2$ by Markov inequality
- So, $Pr[x'[j] \geq x[j] + \varepsilon\ ||x||_1] = Pr[\forall\ k.\ X_{k,j} > \varepsilon\ ||x||_1] \leq 1/2^{\log 1/\delta} = \delta$
- Final result: with certainty $x[j] \leq x'[j]$ and with probability at least $1-\delta$, $x'[j] < x[j] + \varepsilon\ ||x||_1$

L_2 distance

- AMS sketch (for Alon-Matias-Szegedy) proposed in 1996
  - Allows estimation of L_2 (Euclidean) distance between streaming vectors, $||\ x - y\ ||_2$
  - Used at the heart of many streaming and non-streaming mining applications: achieves dimensionality reduction
- Here, describe AMS sketch by generalizing CM sketch.
- Uses extra hash functions $g_1 \ldots g_{\log 1/\delta}\ \{1 \ldots U\} \rightarrow \{+1,-1\}$
- Now, given update $(j,+c)$, set $CM[k,h_k(i)] \mathrel{+}= c^*g_k(j)$



L_2 analysis



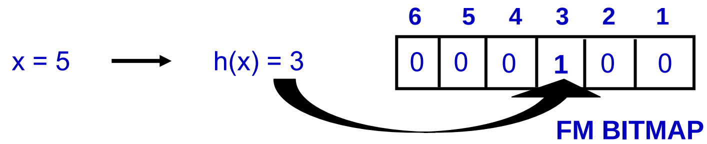
- Estimate $||x||_2^2 = \text{median}_k \sum_i CM[k,i]^2$
- Each row's result is $\sum_k g(i)^2 x_i^2 + \sum_{h(i)=h(j)} 2\, g(i)\, g(j)\, x_i\, x_j$
- But $g(i)^2 = -1^2 = +1^2 = 1$, and $\sum_i x_i^2 = ||x||_2^2$
- $g(i)g(j)$ has 1/2 chance of +1 or –1 : expectation is 0 …

L_2 accuracy

- Formally, one can show an (ε, δ) approximation
  - Expectation of each estimate is exactly $||x||_2^2$ and variance is bounded by ε^2 times expectation squared.
  - Using Chebyshev's inequality, show that probability that each estimate is within $\pm\, \varepsilon\, ||x||_2^2$ is constant
  - Take median of $\log (1/\delta)$ estimates reduces probability of failure to δ (using Chernoff bounds)
- Result: given sketches of size $O(1/\varepsilon^2 \log 1/\delta)$ can estimate $||x||_2^2$ so that result is in $(1\pm\varepsilon)||x||_2^2$ with probability at least $1-\delta$ ❑
  - Note: same analysis used many time in data streams
- In Practice: Can be very fast, very accurate!
  - Used in Sprint 'CMON' tool

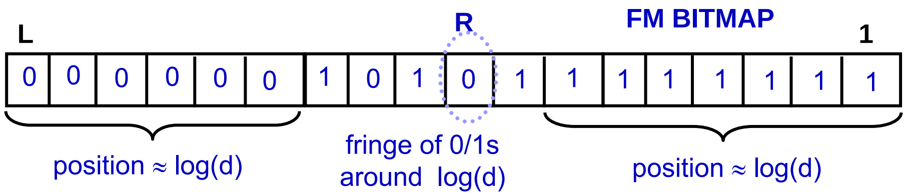
FM Sketch

- Estimates number of distinct inputs (**`count distinct`**)
- Uses hash function mapping input items to i with prob 2^{-i}
  - i.e. Pr[h(x) = 1] = ½, Pr[h(x) = 2] = ¼, Pr[h(x)=3] = 1/8 …
  - Easy to construct h() from a uniform hash function by counting trailing zeros
- Maintain FM Sketch = bitmap array of L = log U bits
  - Initialize bitmap to all 0s
  - For each incoming value x, set FM[h(x)] = 1



FM Analysis

- If d distinct values, expect d/2 map to FM[1], d/4 to FM[2]…



  - Let R = position of rightmost zero in FM, indicator of log(d)
  - Basic estimate $d = c2^R$ for scaling constant $c \approx 1.3$
  - Average many copies (different hash fns) improves accuracy
- With $O(1/\varepsilon^2 \log 1/\delta)$ copies, get (ε,δ) approximation
  - 10 copies gets ≈ 30% error, 100 copies < 10% error

Sketching and Sampling Summary

- Sampling and sketching ideas are at the heart of many stream mining algorithms
  - Entropy computation, association rule mining, clustering (still to come)
- A sample is a quite general representative of the data set; sketches tend to be specific to a particular purpose
  - FM sketch for count distinct, AMS sketch for L_2 estimation

Practicality

- Algorithms discussed here are quite simple and very fast
  - Sketches can easily process millions of updates per second on standard hardware
  - Limiting factor in practice is often I/O related
- Implemented in several practical systems:
  - AT&T's Gigascope system on live network streams
  - Sprint's CMON system on live streams
  - Google's log analysis
- Sample implementations available on the web
  - `http://www.cs.rutgers.edu/~muthu/massdal-code-index.html`
  - or web search for 'massdal'

Other Streaming Algorithms

Many fundamentals have been studied, not covered here:

- Different streaming data types
  - Permutations, Graph Data, Geometric Data (Location Streams)
- Different streaming processing models
  - Sliding Windows, Exponential and other decay, Duplicate sensitivity, Random order streams, Skewed streams
- Different streaming scenarios
  - Distributed computations, sensor network computations

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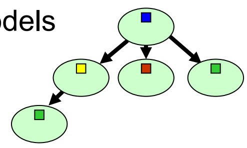
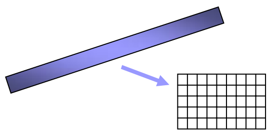
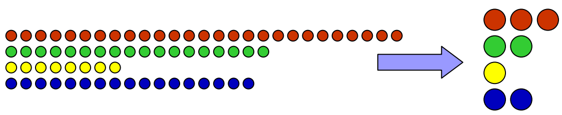
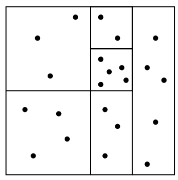





Data Mining on Streams

- Pattern finding: finding common patterns or features
  - Association rule mining, Clustering, Histograms, Wavelet & Fourier Representations
- Data Quality Issues
  - Change Detection, Data Cleaning, Anomaly detection, Continuous Distributed Monitoring
- Learning and Predicting
  - Building Decision Trees, Regression, Supervised Learning
- Putting it all together: Systems Issues
  - Adaptive Load Shedding, Query Languages, Planning and Execution

Association Rule Mining

- Classic example: supermarket wants to discover correlations in buying patterns [Agrawal, Imielinski, Swami 93]
  - (bogus) result: diapers → beer
- **Input**: transactions t_1 = {eggs, milk, bread}, t_2 = {milk} ...t_n
- **Output**: rules of form {eggs, milk} → bread
  - Support: proportion of input containing {eggs, milk, bread}
  - Confidence: $\frac{\text{proportion containing \{eggs, milk, bread\}}}{\text{proportion containing \{eggs, milk\}}}$
- **Goal**: find rules with support, confidence above threshold

Frequent Itemsets

- Association Rule Mining (ARM) algorithms first find all frequent itemsets: subsets of items with support > ϕ
  - m-itemset: itemset with size m, i.e. |X| = m
- Use these frequent itemsets to generate the rules
- Start by finding all frequent 1-itemsets
  - Even this is a challenge in massive data streams

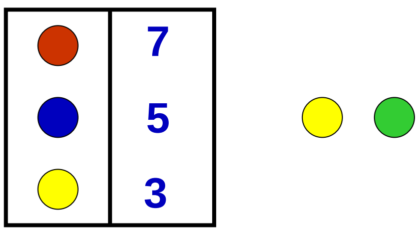
Heavy Hitters Problem

- The 'heavy hitters' are the frequent 1-itemsets
- Many, many streaming algorithms proposed:
  - Random sampling
  - Lossy Counting [Manku, Motwani 02]
  - Frequent [Misra, Gries 82, Karp et al 02, Demaine et al 02]
  - Count-Min, Count Sketches [Charikar, Chen, Farach-Colton 02]
  - And many more...
- 1-itemsets used to find, e.g heavy users in a network
  - The basis of general frequent itemset algorithms
  - A non-uniform kind of sampling

Space Saving Algorithm

- "SpaceSaving" algorithm [Metwally, Agrawal, El Abaddi 05] merges 'Lossy Counting' and 'Frequent' algorithms
  - Gets best space bound, very fast in practice
- Finds all items with count $\geq \phi n$, none with count $< (\phi - \varepsilon)n$
  - Error $0 < \varepsilon < 1$, e.g. $\varepsilon = 1/1000$
  - Equivalently, estimate each frequency with error $\pm\varepsilon n$
- Simple data structure:
  - Keep $k = 1/\varepsilon$ item names and counts, initially zero
  - Fill counters by counting first k distinct items exactly

SpaceSaving Algorithm



- On seeing new item:
  - If it has a counter, increment counter
  - If not, replace item with least count, increment count

SpaceSaving Analysis

- Smallest counter value, min, is at most εn
  - Counters sum to n by induction
  - $1/\varepsilon$ counters, so average is εn: smallest cannot be bigger
- True count of an uncounted item is between 0 and min
  - Proof by induction, true initially, min increases monotonically
  - Hence, the count of any item stored is off by at most εn
- Any item x whose true count $> \varepsilon n$ is stored
  - By contradiction: x was evicted in past, with count $\leq min_t$
  - Every count is an overestimate, using above observation
  - So est. count of $x > \varepsilon n \geq min \geq min_t$, and would not be evicted

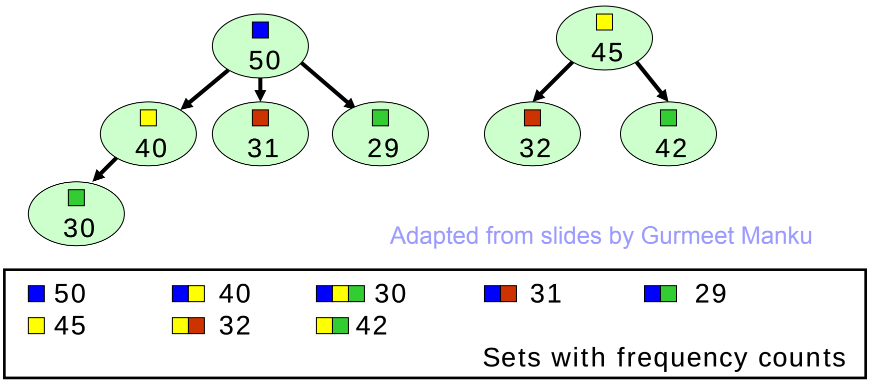
So: Find all items with count $> \varepsilon n$, error in counts $\leq \varepsilon n$

Extending to Frequent Itemsets

- Use similar "approximate counting" ideas for finding frequent itemsets [Manku, Motwani 02]
  - From each new transaction, generate all subsets
  - Track potentially frequent itemsets, prune away infrequent
  - Similar guarantees: error in count at most εn
- Efficiency concerns:
  - Buffer as many transactions as possible, generate subsets together so can prune early
  - Need compact representation of itemsets

Trie Representation of subsets

Compact representation of itemsets in lexicographic order.



Use '*a priori*' rule: if a subset is infrequent, so are all of its supersets – so whole subtrees can be pruned

ARM Summary

- [Manku, Motwani 02] gives details on when and how to prune
- Final Result: can monitor and extract association rules from frequent item sets with high accuracy
- Many extensions and variations to study:
  - Space required depends a lot on input, can be many potential frequent itemsets
  - How to mine when itemsets are observed over many sites (e.g. different routers; stores) and guarantee discovery?
  - Variant definitions: frequent subsequences, sequential patterns, maximal itemsets etc.
  - Sessions later in workshop…

Outline

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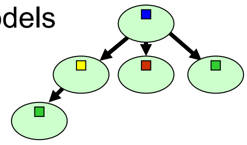
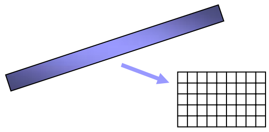
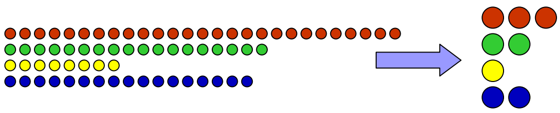
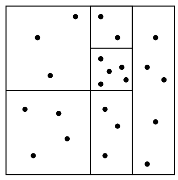
   - Motivating examples, applications and models
   - Random sampling: reservoir and minwise
     - Application: Estimating entropy
   - Sketches: Count-Min, AMS, FM
2. Stream Data Mining Algorithms
   - Association Rule Mining
   - **Change Detection**
   - Clustering









Change Detection

Basic question: monitor a stream of events (network, power grid, sensors etc.), detect "changes" for:
- Anomaly detection – trigger alerts/alarms
- Data cleaning – detect errors in data feeds
- Data mining – indicate when to learn a new model

- What is "change"?
  - Change in behaviour of some subset of items
  - Change in patterns and rules detected
  - Change in underlying distribution of frequencies

Approaches to Change Detection

General idea: compare a reference distribution to a current window of events

- Item changes: individual items with big frequency change
  - Techniques based on sketches
- Fix a distribution (eg. mixture of gaussians), fit parameters
  - Not always clear which distribution to fix *a priori*
- Non-parametric change detection
  - Few parameters to set, but must specify when to call a change significant

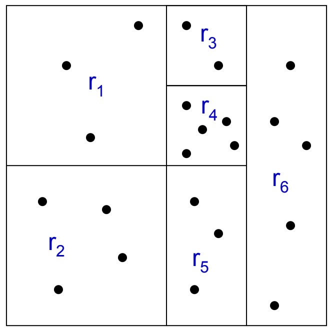
Non-parametric Change Detection

Technique due to [Dasu et al 06]

- Measure change using Kullback-Leibler divergence (KL)
  - Standard measure in statistics
  - Many desirable properties, generalizes t-test and χ^2
- KL divergence = $D(p||q) = \Sigma_x\, p(x) \log_2 p(x)/q(x)$
  - for probability distributions p, q
  - If p, q are distributions over high dimensional spaces, no intersection between samples – need to capture density

Space Division Approach

- Use a hierarchical space division (kd-tree) to define r regions r_i of (approximately equal) weight for the reference data
- Compute discrete probability p over the regions
- Apply same space division over a window of recent stream items to create q
- Compute KL divergence D(p||q)



49 Fundamentals of Analyzing and Mining Data Streams

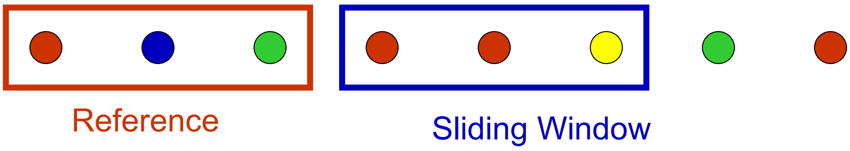
Bootstrapping

How to tell if the KL divergence is significant?

- Statistical bootstrapping approach: use the input data to compute a distribution of distances
- Pool reference and first sliding window data, randomly split into two pieces, measure KL divergence
- Repeat k times, find e.g. 0.99 quantile of divergences
- If KL distance between reference and window > 0.99 quantile of distances for several steps, declare “change”

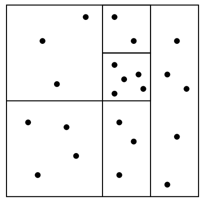
50 Fundamentals of Analyzing and Mining Data Streams

Streaming Computation



For each update:

- Slide window, update region counts
- Update KL divergence between reference p and window q, size w
- Test for significance



Efficient Implementation

- Don't have to recompute KL divergence from scratch
  - Can write normalized KL divergence in terms of

$$\Sigma_i \left(p(r_i) + 1/(2w)\right) \log \frac{p(r_i) + 1/(2w)}{q(r_i) + 1/(2w)}$$

  - Only two terms change per update
- Total time cost per update:
  - Update two regional counts in kd-tree, O(log w)
  - Update KL divergence, in time O(1)
  - Compare to stored divergence cut off for significance test
  - Overall, O(log w)
- Space cost: store tree and counts, O(w)

Change Detection Summary

- Proposed technique is pretty efficient in practice
  - Competitive in accuracy with custom, application-aware change detection
  - Pretty fast – tens of microseconds per update
  - Produces simple description of change based on regions
- Extensions and open problems:
  - Other approaches – histogram or kernel based?
  - Better bootstrapping: quantile approach is only first order accurate…

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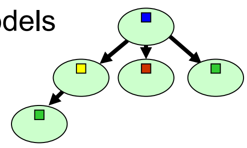
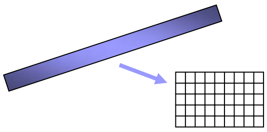
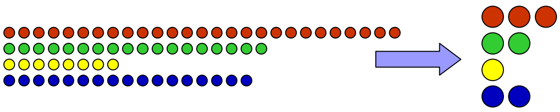
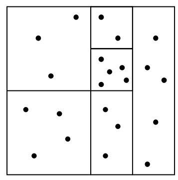


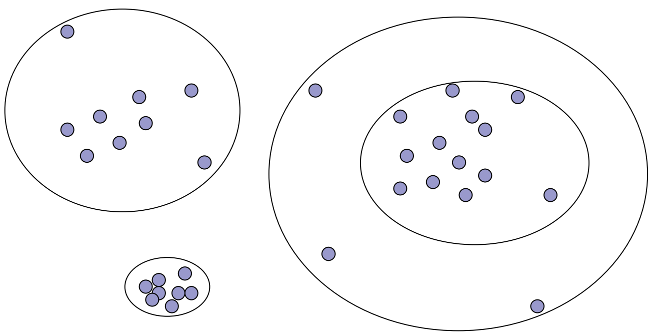




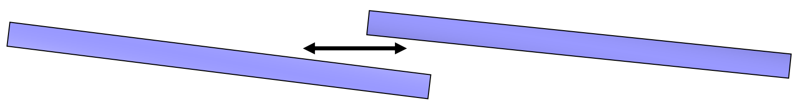


Clustering Data Streams

- We often talk informally about "clusters": 'cancer clusters', 'disease clusters' or 'crime clusters'
- Clustering has an intuitive appeal. We see a bunch of items... we want to discover the clusters...



Stream Clustering Large Points

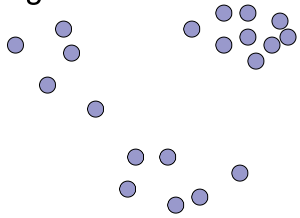


For clustering, need to compare the points. What happens when the points are very high dimensional?
- Eg. trying to compare whole genome sequences
- comparing yesterday's network traffic with today's
- clustering huge texts based on similarity

- If each point is size d, d very large， cost is very high (at least O(d). O(d^2) or worse for some metrics)
- We can do better: create a sketch for each point
- Do clustering using sketched approximate distances

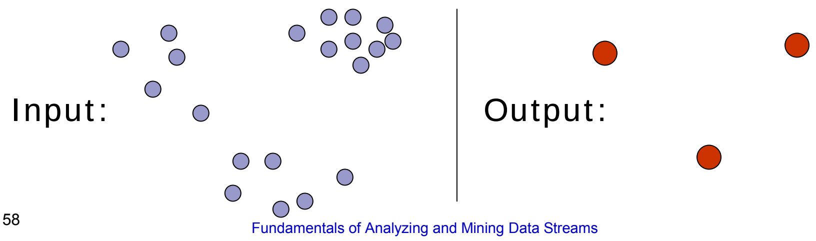
Stream Clustering Many Points

- What does it mean to cluster on the stream when there are too many points to store?
- We see a sequence of points one after the other, and we want to output a clustering for this observed data.
- Moreover, since this clustering changes with time, for each update we maintain some summary information, and at any time can output a clustering.
- Data stream restriction: data is assumed too large to store, so we do not keep all the input, or any constant fraction of it.



Clustering for the stream

- What should output of a stream clustering algorithm be?
- Classification of every input point?
  Too large to be useful?
  Might this change as more input points arrive?
  - Two points which are initially put in different clusters might end up in the same one
- An alternative is to output k cluster centers at end
  - any point can be classified using these centers.



Approximation for k-centers

k-center: minimize diameter (max dist) of each cluster.

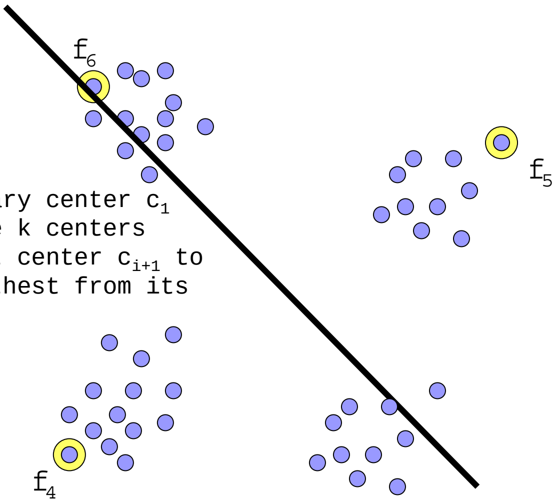
- Pick some point from the data as the first center.
  Repeat:
  - For each data point, compute distance d_{min} from its closest center
  - Find the data point that maximizes d_{min}
  - Add this point to the set of centers

  Until k centers are picked

- If we store the current best center for each point, then each pass requires O(1) time to update this for the new center, else O(k) to compare to k centers.
- So time cost is O(kn), but k passes [Gonzalez, 1985].

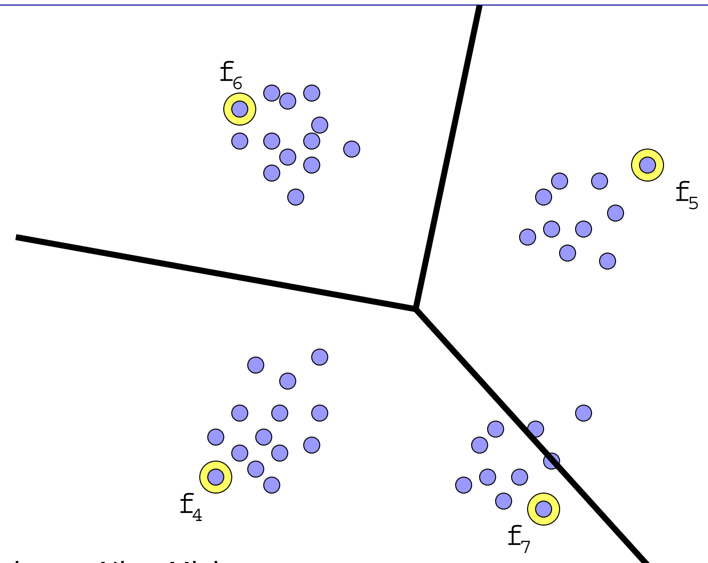
Gonzalez Clustering k=4

ALG:
Select an arbitrary center c_1
Repeat until have k centers
  Select the next center c_{i+1} to be the one farthest from its closest center



Slide due to Nina Mishra

Gonzalez Clustering k=4



Slide due to Nina Mishra

Gonzalez Clustering k=4

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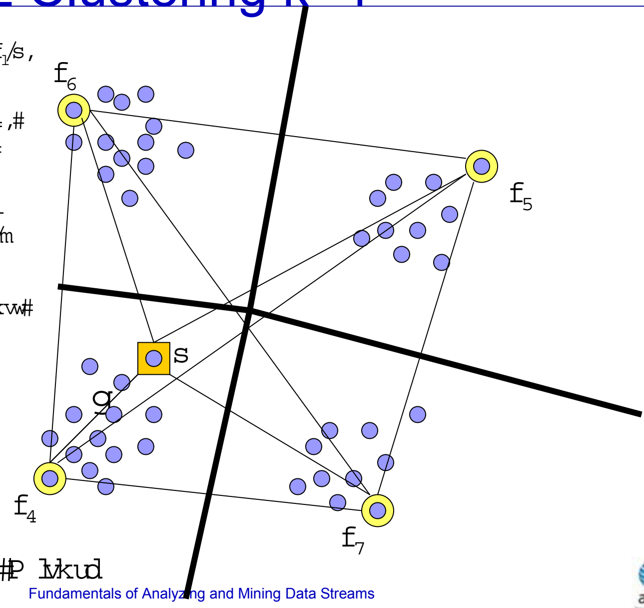

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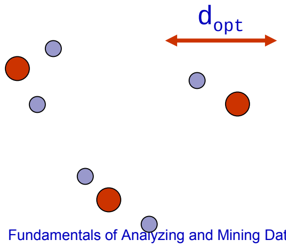
Gonzalez is 2-approximation

- After picking k points to be centers, find next point that would be chosen. Let distance from closest center = d_{opt}
- We have k+1 points, every pair is separated by at least d_{opt}. Any clustering into k sets must put some pair in same set, so any k-clustering must have diameter d_{opt}
- For any two points allocated to the same center, they are both at distance at most d_{opt} from their closest center
- Their distance is at most $2d_{opt}$, using triangle inequality.
- Diameter of any clustering must be at least d_{opt}, and is at most $2d_{opt}$ – so we have a 2 approximation.
- Lower bound: NP-hard to *guarantee* better than 2

Gonzalez Restated

- Suppose we knew d_{opt} (from Gonzalez algorithm for k-centers) at the start
- Do the following procedure:
- `Select the first point as the first center`
- `For each point that arrives:`
  - `Compute` d_{min}, `the distance to the closest center`
  - `If` $d_{min} > d_{opt}$ `then set the new point to be a new center`



Analysis Restated

- d_{opt} is given, so we know that there are k+1 points separated by $\geq d_{opt}$ and d_{opt} is as large as possible
- So there are $\leq$ k points separated by $> d_{opt}$
- New algorithm outputs at most k centers: only include a center when its distance is $> d_{opt}$ from all others. If $>$ k centers output, then $>$ k points separated by $> d_{opt}$, contradicting optimality of d_{opt}.
- Every point not chosen as a center is $< d_{opt}$ from some center and so at most $2d_{opt}$ from any point allocated to the same center (triangle inequality)
- So: given d_{opt} we find a clustering where every point is at most twice this distance from its closest center

Guessing the optimal solution

- Hence, a 2-approximation – but, we aren't given d_{opt}
  - If we knew $d < d_{opt} < 2d$ then we could run the algorithm. If we find more than k centers, we guessed d_{opt} too low
  - So, in parallel, guess d_{opt} = 1, 2, 4, 8...
  - We reject everything $< d_{opt}$, so best guess is $< 2d_{opt}$: our output will be $< 2*2d_{opt}/d_{opt} = 4$ approx
- Need $\log_2 (d_{max}/d_{smallest})$ guesses, $d_{smallest}$ is minimum distance between any pair of points, as $d_{smallest} < d_{opt}$
- $O(k \log(d_{max} / d_{smallest}))$ may be high, can we reduce more?
- [Charikar et al 97]: doubling alg uses only O(k) space, gives 8-approximation. Subsequent work studied other settings

Clustering Summary

- General techniques: keeping small subset ("core-set") of input; guessing a key value; combining subproblems
- Many more complex solutions from computational geometry
- Variations and extensions:
  - When few data points but data points are high dimensional, use sketching techniques to represent
  - Different objectives: k-median, k-means, etc.
  - Better approximations, different guarantees (e.g. outputs 2k clusters, quality as good as that of best k-clustering)

Summary

- We have looked at
  - Sampling from streams and applications (entropy)
  - Sketch summaries for more advanced computations
  - Association Rule Mining to find interesting patterns
  - Change Detection for anomaly detection and alerts
  - Clustering to pick out significant clusters
- Many other variations to solve the problems discussed here, many other problems to study on data streams
  - See more over the course of this workshop.
  - Other tutorials and surveys: [Muthukrishnan '05] [Garofalakis, Gehrke, Rastogi '02]

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